Control Engineering
Dr. Ibrahim Hamarash

Sheet # 2
Root locus technique

Question. sketch the complete root locus for the system having

$$G(s)H(s) = \frac{K}{s(s+3)(s^2+3s+3)}$$

Solution.

step1. $n=4$, $m=0$, no. of branches $=4$

step2. Start points at: $0, -3, -1.5 \pm j0.866$

step3. Angle of asymptotes:

$\theta_0 = 45^\circ$, $\theta_1 = 135^\circ$, $\theta_2 = 225^\circ$, $\theta_3 = 315^\circ$

step 4. Centroid,

$$\sigma = \frac{\sum \text{Poles} - \sum \text{Zeros}}{n-m} = \frac{0-3-1.5-1.5-0}{4-0} = -1.5$$

step5. Breakaway points,

the characteristic equation is

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(s+3)(s^2+3s+3)} = 0$$

$$s^4 + 6s^3 + 12s^2 + 9s + K = 0$$

$$\frac{d}{ds}(\text{charact. eqn.}) = 4s^3 + 18s^2 + 24s + 9 = 0$$

by trial and error

test $s = -1.5$
$s = -0.63$
$s = -2.366$

because the equation is 3rd order, we can test three values, sometime we use the long division method.

PAGE:
DATE:

Step 6. Intersection with imaginary axis,

$$s^4 + 6s^3 + 12s^2 + 9s + k = 0$$

s^4	1	12	k
s^3	6	9	0
s^2	10.5	k	0
s^1	$\frac{94.5 - 6k}{10.5}$	0	0
s^0	k		

$$94.5 - 6k = 0 \Rightarrow k = 15.75$$

Aux eqn is: $10.5s^2 + k = 0$

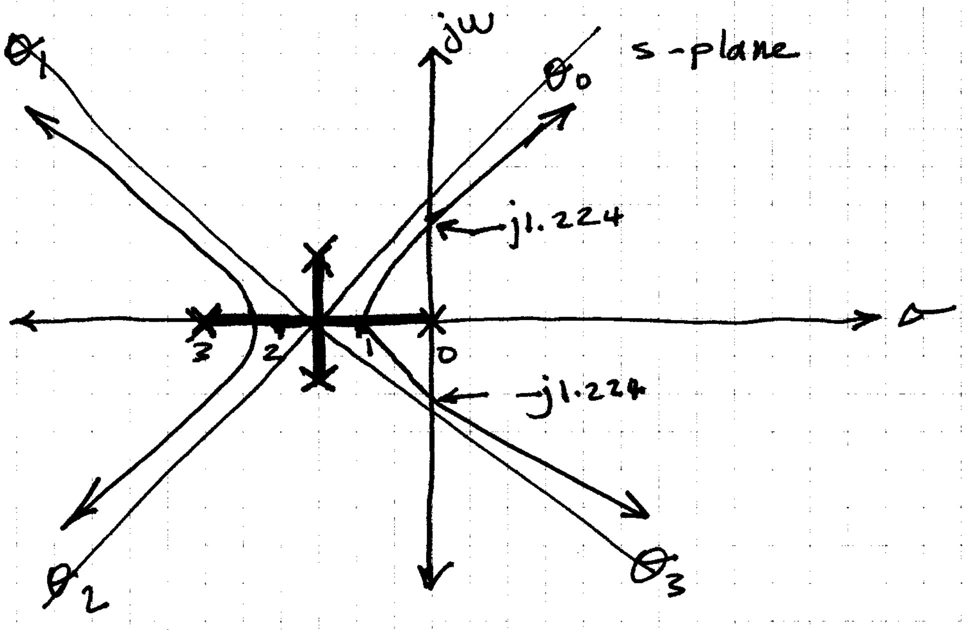
$$10.5s^2 + 15.75 = 0 \Rightarrow s^2 = -1.5$$

$$\therefore s = \pm j1.224$$

Step 7. Angle of departure

$$\theta_d \,(\text{at } s = -1.5 + j0.866) = -90^\circ$$

$$\therefore \theta_d \,(\text{at } s = -1.5 - j0.866) = 90^\circ$$



Comment: For $0 < k < 15.75$, system is stable
at $k = 15.75$, system is marginally stable
$k > 15.75$, system is unstable